

## INTEGRÁLSZÁMÍTÁS

### Alapintegrálok

- $\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$
- $\int \frac{1}{x} dx = \ln |x| + C$
- $\int \sin x dx = -\cos x + C$
- $\int \cos x dx = \sin x + C$
- $\int \frac{1}{\cos^2 x} dx = \operatorname{tg} x + C$
- $\int \frac{1}{\sin^2 x} dx = -\operatorname{ctg} x + C$
- $\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + C$
- $\int \frac{1}{1+x^2} dx = \operatorname{arctg} x + C$
- $\int e^x dx = e^x + C$
- $\int a^x dx = \frac{a^x}{\ln a} + C \quad (0 < a \neq -1)$

### Alapintegrálokra visszavezethető feladatok

- $\int (x^5 + \frac{1}{x^3}) dx = \int (x^5 + x^{-3}) dx = \frac{x^6}{6} + \frac{x^{-2}}{-2} + C$
- $\int \left( \sqrt{x} + \frac{1}{\sqrt{x}} \right) dx = \int (x^{\frac{1}{2}} + x^{-\frac{1}{2}}) dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C = \frac{2}{3} \sqrt{x^3} + 2\sqrt{x} + C$
- $\int \left( \sqrt[3]{x^2} + \frac{1}{\sqrt{x}} \right) dx = \int (x^{\frac{2}{3}} + x^{-\frac{1}{2}}) dx = \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + \frac{x^{\frac{3}{4}}}{\frac{3}{4}} + C = \frac{3}{5} \sqrt[3]{x^5} + \frac{4}{3} \sqrt[4]{x^3} + C$
- $\int \sqrt{x} \cdot \sqrt[3]{x} dx = \int x^{\frac{1}{2}} \cdot x^{\frac{1}{3}} dx = \int x^{\frac{3+2}{6}} dx = \int x^{\frac{5}{6}} dx = \frac{x^{\frac{11}{6}}}{\frac{11}{6}} + C = \frac{6}{11} \sqrt[6]{x^{11}} + C$
- $\int \frac{\sqrt{x}}{\sqrt[3]{x}} dx = \int \frac{x^{\frac{1}{2}}}{x^{\frac{1}{3}}} dx = \int x^{\frac{3-2}{6}} dx = \int x^{\frac{1}{6}} dx = \frac{x^{\frac{7}{6}}}{\frac{7}{6}} + C = \frac{6}{7} \sqrt[6]{x^7} + C$
- $\int \sqrt{x} \cdot \sqrt[3]{x} dx = \int (x \cdot x^{\frac{1}{3}})^{\frac{1}{2}} dx = \int (x^{\frac{4}{3}})^{\frac{1}{2}} dx = \int x^{\frac{2}{3}} dx = \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + C = \frac{3}{5} \sqrt[3]{x^5} + C$
- $\int \frac{x^2 - 7x + 8}{x^2} dx = \int \left( 1 - \frac{7}{x} + \frac{8}{x^2} \right) dx = x - 7 \ln |x| - \frac{8}{x} + C$
- $\int \frac{x^3 - 4x^2}{\sqrt{x}} dx = \int \frac{x^3 - 4x^2}{x^{\frac{1}{2}}} dx = \int (x^{\frac{5}{2}} - 4x^{\frac{3}{2}}) dx = \frac{x^{\frac{7}{2}}}{\frac{7}{2}} - 4 \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + C = \frac{2}{7} \sqrt{x^7} - 4 \cdot \frac{2}{5} \cdot \sqrt{x^5} + C$
- $\int (5 \cdot 2^x + 4 \sin x - 3 \cos x) dx = 5 \cdot \frac{2^x}{\ln 2} - 4 \cos x - 3 \sin x + C$

$$10. \int \left( \frac{3}{\cos^2 x} - \frac{7}{5 \sin^2 x} \right) dx = 3 \operatorname{tg} x + \frac{7}{5} \operatorname{ctg} x + C$$

$$11. \int \frac{-4}{3+3x^2} dx = -\frac{4}{3} \int \frac{1}{1+x^2} dx = -\frac{4}{3} \operatorname{arctg} x + C$$

$$12. \int \frac{x^2}{x^2+1} dx = \int \frac{x^2+1-1}{x^2+1} dx = \int \left( 1 - \frac{1}{x^2+1} \right) dx = x - \operatorname{arctg} x + C$$

$$13. \int \frac{5}{3\sqrt{4-4x^2}} dx = \frac{5}{3 \cdot 2} \int \frac{1}{\sqrt{1-x^2}} dx = \frac{5}{6} \operatorname{arcsin} x + C$$

$$14. \int \operatorname{tg}^2 x dx = \int \frac{\sin^2 x}{\cos^2 x} dx = \int \frac{1-\cos^2 x}{\cos^2 x} dx = \int \left( \frac{1}{\cos^2 x} - 1 \right) dx = \operatorname{tg} x - x + C$$

$$15. \int \frac{\cos^2 x - 5}{1 + \cos 2x} dx = \int \frac{\cos^2 x - 5}{(\sin^2 x + \cos^2 x) + (\cos^2 x - \sin^2 x)} dx = \int \frac{\cos^2 x - 5}{2 \cos^2 x} dx =$$

$$= \int \left( \frac{1}{2} - \frac{5}{2 \cos^2 x} \right) dx = \frac{1}{2} x - \frac{5}{2} \operatorname{tg} x + C$$

$$16. \int \frac{\cos 2x}{\sin x + \cos x} dx = \int \frac{\cos^2 x - \sin^2 x}{\sin x + \cos x} dx = \int \frac{(\cos x - \sin x) \cdot (\cos x + \sin x)}{\sin x + \cos x} dx =$$

$$= \int (\cos x - \sin x) dx = \sin x + \cos x + C$$

### Integrálási módszerek

$$\text{I. } \int f(ax+b) dx = \frac{F(ax+b)}{a}, \text{ ahol } F' = f.$$

#### Feladatok

$$1. \text{ a) } \int (2x+9)^3 dx = \frac{(2x+9)^4}{4 \cdot 2} + C$$

$$1. \text{ b) } \int (11x-12)^{13} dx = \frac{(11x-12)^{14}}{11 \cdot 14} + C$$

$$2. \int \frac{1}{(7x+1)^4} dx = \int (7x+1)^{-4} dx = \frac{(7x+1)^{-3}}{-3 \cdot 7} + C$$

$$3. \text{ a) } \int \sqrt{5x-8} dx = \int (5x-8)^{\frac{1}{2}} dx = \frac{(5x-8)^{\frac{3}{2}}}{\frac{3}{2} \cdot 5} + C$$

$$3. \text{ b) } \int \sqrt[3]{1-2x} dx = \int (1-2x)^{\frac{1}{3}} dx = \frac{(1-2x)^{\frac{4}{3}}}{\frac{4}{3} \cdot (-2)} + C \quad 3. \text{ c) } \int (4x+2)^{\frac{2}{3}} dx = \frac{(4x+2)^{\frac{5}{3}}}{\frac{5}{3} \cdot 4} + C$$

$$4. \int \frac{1}{\sqrt{3-4x}} dx = \int (3-4x)^{-\frac{1}{2}} dx = \frac{(3-4x)^{\frac{1}{2}}}{\frac{1}{2} \cdot (-4)} + C$$

$$5. \text{ a) } \int \sin 6x dx = \frac{-\cos 6x}{6} + C$$

$$5. \text{ b) } \int \cos(-4-5x) dx = \frac{\sin(-4-5x)}{-5} + C$$

$$6. \text{ a) } \int \frac{1}{\sin^2\left(2x + \frac{\pi}{4}\right)} dx = -\frac{1}{2} \operatorname{ctg}\left(2x + \frac{\pi}{4}\right) + C$$

$$6. \text{ b) } \int \frac{5}{\cos^2 3x} dx = \frac{5}{3} \operatorname{tg} 3x + C$$

$$7. \text{ a) } \int \frac{1}{1+3x^2} dx = \int \frac{1}{1+(\sqrt{3}x)^2} dx = \frac{\operatorname{arctg}(\sqrt{3}x)}{\sqrt{3}} + C$$

$$7. \text{ b) } \int \frac{-2}{20x^2+5} dx = -\frac{2}{5} \int \frac{1}{4x^2+1} dx = -\frac{2}{5} \int \frac{1}{1+(2x)^2} dx = -\frac{2}{5} \frac{\operatorname{arctg} 2x}{2} + C$$

$$7. \text{ c) } \int \frac{1}{x^2+2x+2} dx = \int \frac{1}{(x+1)^2+1} dx = \operatorname{arctg}(x+1) + C$$

$$8. \int \frac{1}{\sqrt{1-25x^2}} dx = \int \frac{1}{\sqrt{1-(5x)^2}} dx = \frac{\operatorname{arcsin}(5x)}{5} + C$$

$$9. \text{ a) } \int e^{6x+5} dx = \frac{e^{6x+5}}{6} + C$$

$$9. \text{ b) } \int (e^{-x} + e^{-2x}) dx = -e^{-x} - \frac{1}{2} e^{-2x} + C$$

$$10. \int 5^{3-11x} dx = \frac{5^{3-11x}}{-11} \cdot \frac{1}{\ln 5} + C$$

$$11. \text{ a) } \int \frac{1}{8x-3} dx = \frac{\ln |8x-3|}{8} + C$$

$$11. \text{ b) } \int \frac{x+1}{x-1} dx = \int \frac{x-1+2}{x-1} dx = \int \left(1 + \frac{2}{x-1}\right) dx = x + 2 \ln |x-1| + C$$

$$12. \text{ a) } \int \sin^2 x dx = \int \frac{1}{2} (1 - \cos 2x) dx = \int \left(\frac{1}{2} - \frac{\cos 2x}{2}\right) dx = \frac{1}{2} x - \frac{\sin 2x}{2 \cdot 2} + C$$

$$12. \text{ b) } \int \cos^2 x dx = \int \frac{1}{2} (1 + \cos 2x) dx = \int \left(\frac{1}{2} + \frac{\cos 2x}{2}\right) dx = \frac{1}{2} x + \frac{\sin 2x}{2 \cdot 2} + C$$

$$12. c) \int \sin^2 2x dx = \int \frac{1}{2}(1 - \cos 4x) dx = \int \left( \frac{1}{2} - \frac{\cos 4x}{2} \right) dx = \frac{1}{2}x - \frac{\sin 4x}{2 \cdot 4} + C$$

$$\text{II. } \int f^n(x) f'(x) dx = \frac{f^{n+1}(x)}{n+1} + C \quad (n \neq -1).$$

### Feladatok

$$1. a) \int 8x(1+4x^2)^3 dx = \frac{(1+4x^2)^4}{4} + C$$

$$1. b) \int x^2(2x^3+9) dx = \frac{1}{6} \int 6x^2(2x^3+9) dx = \frac{1}{6} \frac{(2x^3+9)^2}{2} + C$$

$$1. c) \int x^2(x^3-2)^5 dx = \frac{1}{3} \int 3x^2(x^3-2)^5 dx = \frac{1}{3} \frac{(x^3-2)^6}{6} + C$$

$$2. a) \int x^2 \sqrt{6x^3+7} dx = \frac{1}{18} \int 18x^2(6x^3+7)^{\frac{1}{2}} dx = \frac{1}{18} \frac{(6x^3+7)^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$2. b) \int x \cdot \sqrt[4]{2-3x^2} dx = -\frac{1}{6} \int -6x(2-3x^2)^{\frac{1}{4}} dx = -\frac{1}{6} \frac{(2-3x^2)^{\frac{5}{4}}}{\frac{5}{4}} + C$$

$$3. a) \int \frac{x}{\sqrt{x^2+1}} dx = \int x(x^2+1)^{-\frac{1}{2}} dx = \frac{1}{2} \int 2x(x^2+1)^{-\frac{1}{2}} dx = \frac{1}{2} \frac{(x^2+1)^{\frac{1}{2}}}{\frac{1}{2}} + C = \sqrt{x^2+1} + C$$

$$3. b) \int \frac{x}{\sqrt{(x^2+1)^3}} dx = \int x(x^2+1)^{-\frac{3}{2}} dx = \frac{1}{2} \int 2x(x^2+1)^{-\frac{3}{2}} dx = \frac{1}{2} \frac{(x^2+1)^{-\frac{1}{2}}}{-\frac{1}{2}} + C = -\frac{1}{\sqrt{x^2+1}} + C$$

$$3. c) \int \frac{7x^2}{\sqrt{5-4x^3}} dx = \int 7x^2(5-4x^3)^{-\frac{1}{2}} dx = -\frac{7}{12} \int -12x^2(5-4x^3)^{-\frac{1}{2}} dx = -\frac{7}{12} \frac{(5-4x^3)^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$3. d) \int \frac{2x-5}{\sqrt[3]{(x^2-5x+13)^7}} dx = \int (2x-5)(x^2-5x+13)^{-\frac{7}{3}} dx = \frac{(x^2-5x+13)^{-\frac{4}{3}}}{-\frac{4}{3}} + C$$

$$4. a) \int \sin x \cos x dx = \int \sin x \cdot (\sin x)' dx = \frac{\sin^2 x}{2} + C$$

$$4. b) \int \cos^3 x \sin x dx = -\int \cos^3 x \cdot (-\sin x) dx = -\frac{\cos^4 x}{4} + C$$

$$4. c) \int \sin^4 x \sin 2x dx = \int \sin^4 x 2 \sin x \cos x dx = 2 \int \sin^5 x \cos x dx = 2 \frac{\sin^6 x}{6} + C = \frac{1}{3} \sin^6 x + C$$

$$4. d) \int \frac{\sin x}{\sqrt[3]{\cos^2 x}} dx = \int \sin x (\cos x)^{-\frac{2}{3}} dx = -\int (-\sin x) (\cos x)^{-\frac{2}{3}} dx = -\frac{(\cos x)^{\frac{1}{3}}}{\frac{1}{3}} + C = -3 \cdot \sqrt[3]{\cos x} + C$$

$$5. \int \sin^3 x dx = \int \sin x \sin^2 x dx = \int \sin x (1 - \cos^2 x) dx = \int (\sin x - \sin x \cos^2 x) dx = -\cos x + \frac{\cos^3 x}{3} + C$$

$$6. a) \int \frac{\ln x}{x} dx = \int \frac{1}{x} \ln x dx = \frac{\ln^2 x}{2} + C$$

$$6. b) \int \frac{\ln^3 x}{x} dx = \int \frac{1}{x} \ln^3 x dx = \frac{\ln^4 x}{4} + C$$

$$6. c) \int \frac{1}{x \ln^3 x} dx = \int \frac{1}{x} \ln^{-3} x dx = \frac{\ln^{-2} x}{-2} + C$$

$$7. a) \int \frac{\sin^5 x}{\cos^7 x} dx = \int \frac{\sin^4 x}{\cos^5 x} \cdot \frac{1}{\cos^2 x} dx = \int \sin^4 x \frac{1}{\cos^2 x} dx = \frac{\tan^6 x}{6} + C$$

$$7. b) \int \frac{1}{\cos^2 x \sqrt{\tan x}} dx = \int \frac{1}{\cos^2 x} (\tan x)^{-\frac{1}{2}} dx = \frac{(\tan x)^{\frac{1}{2}}}{\frac{1}{2}} + C = 2\sqrt{\tan x} + C$$

$$8. \int \frac{(\arctan x)^2}{1+x^2} dx = \int \frac{1}{1+x^2} (\arctan x)^2 dx = \frac{(\arctan x)^3}{3} + C$$

$$9. \int \frac{1}{\sqrt{(1-x^2)} \arcsin x} dx = \int \frac{1}{\sqrt{1-x^2}} (\arcsin x)^{-\frac{1}{2}} dx = \frac{(\arcsin x)^{\frac{1}{2}}}{\frac{1}{2}} + C = 2\sqrt{\arcsin x} + C$$

$$10. \int e^x (1-e^x)^3 dx = -\int -e^x (1-e^x)^3 dx = -\frac{(1-e^x)^4}{4} + C$$

$$\text{III. } \int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + C$$

### Feladatok

$$1. \int \frac{2x}{x^2+7} dx = \ln(x^2+7) + C$$

$$2. \int \frac{5x^2}{x^3+4} dx = \frac{5}{3} \int \frac{3x^2}{x^3+4} dx = \frac{5}{3} \ln |x^3+4| + C$$

$$3. \int \frac{x^3}{x^4+5} dx = \frac{1}{4} \int \frac{4x^3}{x^4+5} dx = \frac{1}{4} \ln(x^4+5) + C$$

$$4. \int \frac{x-3}{x^2-6x+10} dx = \frac{1}{2} \int \frac{2x-6}{x^2-6x+10} dx = \frac{1}{2} \ln(x^2-6x+10) + C$$

$$5. a) \int ctg x dx = \int \frac{\cos x}{\sin x} dx = \ln |\sin x| + C$$

$$5. b) \int tg x dx = \int \frac{\sin x}{\cos x} dx = -\int \frac{-\sin x}{\cos x} dx = -\ln |\cos x| + C$$

$$6. a) \int \frac{\cos x}{3 \sin x + 1} dx = \frac{1}{3} \int \frac{3 \cos x}{3 \sin x + 1} dx = \frac{1}{3} \ln |3 \sin x + 1| + C$$

$$6. b) \int \frac{4 \sin x}{5 \cos x + 4} dx = -\frac{4}{5} \int \frac{-5 \sin x}{5 \cos x + 4} dx = -\frac{4}{5} \ln |5 \cos x + 4| + C$$

$$7. \int \frac{\sin 2x}{\sin^2 x + \pi} dx = \int \frac{2 \sin x \cos x}{\sin^2 x + \pi} dx = \ln(\sin^2 x + \pi) + C$$

$$8. a) \int \frac{1}{\cos^2 x \operatorname{tg} x} dx = \int \frac{1}{\cos^2 x} \frac{1}{\operatorname{tg} x} dx = \ln |\operatorname{tg} x| + C$$

$$8. b) \int \frac{1}{\sin^2 x \operatorname{ctg} x} dx = -\int \frac{1}{\sin^2 x} \frac{1}{\operatorname{ctg} x} dx = -\ln |\operatorname{ctg} x| + C$$

$$9. \int \frac{1}{(1+x^2) \operatorname{arctg} x} dx = \int \frac{1}{1+x^2} \frac{1}{\operatorname{arctg} x} dx = \ln |\operatorname{arctg} x| + C$$

$$10. \int \frac{1}{\sqrt{1-x^2} \arcsin x} dx = \int \frac{1}{\sqrt{1-x^2}} \frac{1}{\arcsin x} dx = \ln |\arcsin x| + C$$

$$11. \int \frac{1}{x \ln x} dx = \int \frac{1}{x} \frac{1}{\ln x} dx = \ln |\ln x| + C$$

$$12. \int \frac{e^{2x}}{e^{2x} + 3} dx = \frac{1}{2} \int \frac{2e^{2x}}{e^{2x} + 3} dx = \frac{1}{2} \ln(e^{2x} + 3) + C$$

#### IV. Parciális integrálás

$$\int u'(x) \cdot v(x) dx = u(x)v(x) - \int u(x)v'(x) dx$$

#### Feladatok

$$1. \int \underset{v}{x} \cdot \underset{u'}{e^x} dx = x \cdot e^x - \int e^x dx = x \cdot e^x - e^x + C$$

$$2. \int \underset{v}{x} \cdot \underset{u'}{e^{2x}} dx = x \cdot \frac{e^{2x}}{2} - \int \frac{e^{2x}}{2} dx = x \cdot \frac{e^{2x}}{2} - \frac{e^{2x}}{4} + C$$

$$3. \int x^2 \cdot e^{2x+4} dx = x^2 \cdot \frac{e^{2x+4}}{2} - \int 2x \cdot \frac{e^{2x+4}}{2} dx = x^2 \cdot \frac{e^{2x+4}}{2} - \left[ 2x \cdot \frac{e^{2x+4}}{4} - \int 2 \cdot \frac{e^{2x+4}}{4} dx \right] =$$

$$= x^2 \cdot \frac{e^{2x+4}}{2} - 2x \cdot \frac{e^{2x+4}}{4} + 2 \cdot \frac{e^{2x+4}}{8} + C$$

$$4. \int x \cdot \sin 5x dx = x \cdot \left( -\frac{\cos 5x}{5} \right) - \int -\frac{\cos 5x}{5} dx = -x \cdot \frac{\cos 5x}{5} + \frac{\sin 5x}{25} + C$$

$$5. \int x^2 \cdot \cos x dx = x^2 \sin x - \int 2x \cdot \sin x dx = x^2 \sin x - \left[ 2x(-\cos x) - \int 2 \cdot (-\cos x) dx \right] =$$

$$= x^2 \sin x + 2x \cos x - 2 \sin x + C$$

$$6. x \in \mathbb{R}^+, \int \ln x dx = \int 1 \cdot \ln x dx = x \ln x - \int x \cdot \frac{1}{x} dx = x \ln x - x + C$$

$$7. x \in ]-1, 1[, \int \arcsin x dx = \int 1 \cdot \arcsin x dx = x \arcsin x - \int x \frac{1}{\sqrt{1-x^2}} dx =$$

$$= x \arcsin x + \frac{1}{2} \int -2x \cdot (1-x^2)^{-\frac{1}{2}} dx = x \arcsin x + \frac{1}{2} \frac{(1-x^2)^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$= x \arcsin x + \sqrt{1-x^2} + C$$

$$8. \int \operatorname{arctg} 3x dx = \int 1 \cdot \operatorname{arctg} 3x dx = x \operatorname{arctg} 3x - \int x \cdot \frac{1}{1+9x^2} \cdot 3 dx =$$

$$= x \operatorname{arctg} 3x - \frac{1}{6} \int 18x \cdot \frac{1}{1+9x^2} dx = x \operatorname{arctg} 3x - \frac{1}{6} \ln(1+9x^2) + C$$

$$9. x \in \left] -\frac{3}{5}, \infty \right[, \int x \cdot \sqrt{5x+3} dx =$$

$$= x \cdot \frac{(5x+3)^{\frac{3}{2}}}{\frac{3}{2} \cdot 5} - \int 1 \cdot \frac{(5x+3)^{\frac{3}{2}}}{\frac{3}{2} \cdot 5} dx = \frac{2}{15} x \cdot (5x+3)^{\frac{3}{2}} - \frac{2}{15} \cdot \frac{(5x+3)^{\frac{5}{2}}}{\frac{5}{2} \cdot 5} + C$$

$$10. x \in \mathbb{R}^+, \int \frac{\ln x}{\sqrt{x}} dx = \int \frac{1}{\sqrt{x}} \cdot \ln x dx = \frac{x^{\frac{1}{2}}}{\frac{1}{2}} \ln x - \int \frac{x^{\frac{1}{2}}}{\frac{1}{2}} \cdot \frac{1}{x} dx =$$

$$= 2x^{\frac{1}{2}} \ln x - \int 2x^{-\frac{1}{2}} dx = 2x^{\frac{1}{2}} \ln x - 2 \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C = 2\sqrt{x} \ln x - 4\sqrt{x} + C$$

$$11. \int \ln(x^2+1) dx = \int 1 \cdot \ln(x^2+1) dx = x \ln(x^2+1) - \int x \cdot \frac{1}{x^2+1} \cdot 2x dx =$$

$$= x \ln(x^2+1) - \int \frac{2x^2+2-2}{x^2+1} dx = x \ln(x^2+1) - \int \left( 2 - \frac{2}{x^2+1} \right) dx =$$

$$= x \ln(x^2+1) - 2x + 2 \operatorname{arctg} x + C$$

V.  $\int f(g(x)) \cdot g'(x) dx = F(g(x)) + C$ , ahol  $F' = f$ .

### Feladatok

1. a)  $\int 2x \cos(x^2 + 1) dx = \sin(x^2 + 1) + C$

1. b)  $\int x \sin x^2 dx = -\frac{1}{2} \cos x^2 + C$

1. c)  $\int (3x^2 + 2) \sin(x^3 + 2x - 4) dx = -\cos(x^3 + 2x - 4) + C$

2. a)  $\int e^{\sin x} \cos x dx = e^{\sin x} + C$

2. b)  $\int \frac{1}{\cos^2 x} e^{tgx} dx = e^{tgx} + C$

2. c)  $\int x e^{-x^2} dx = -\frac{1}{2} \int -2x e^{-x^2} dx = -\frac{1}{2} e^{-x^2} + C$

3. a)  $\int \frac{2x}{\sqrt{1-x^4}} dx = \int 2x \frac{1}{\sqrt{1-(x^2)^2}} dx = \arcsin x^2 + C$

3. b)  $\int \frac{e^x}{\sqrt{1-e^{2x}}} dx = \int e^x \frac{1}{\sqrt{1-(e^x)^2}} dx = \arcsin(e^x) + C$

4. a)  $\int \frac{2x}{1+x^4} dx = \int 2x \frac{1}{1+(x^2)^2} dx = \operatorname{arctg} x^2 + C$

4. b)  $\int \frac{e^x}{1+e^{2x}} dx = \operatorname{arctg}(e^x) + C$

4. c)  $\int \frac{1}{x(1+\ln^2 x)} dx = \int \frac{1}{x} \cdot \frac{1}{1+(\ln x)^2} dx = \operatorname{arctg}(\ln x) + C$

4. d)  $\int \frac{\cos x}{1+\sin^2 x} dx = \operatorname{arctg}(\sin x) + C$